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# Optimizing Basel III Liquidity Coverage Ratios

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## 1. Introduction

The recent financial crises, viz., the subprime mortgage and 2007-2009 financial crises as well as the sovereign debt crisis, are characterized by too-big-to fail banks that suffered from a lack of liquidity (see, for instance, [13] and [14]). The actualization of such liquidity risk led to credit crunches and had negative effects on global financial markets. In response to this, among other things, the Basel Committee on Banking Supervision (BCBS) is proposing that banks should always have a 30-day liquidity cover for stress scenarios (see, for instance, [2], [3], [4] and [5]).

In this regard, the level of high-quality liquid assets (HQLAs) is important in order for banks to function optimally (see, for instance, [6] and [8]). As far as Basel III liquidity proposals are concerned, the BCBS is suggesting a liquidity coverage ratio (LCR) defined as follows.

The LCR has two components:

(a) total stock of HQLAs; and

(b) total nett cash outflows,

and is expressed as

$$\text{LCR} = \frac{\text{Total Stock of High-Quality Liquid Assets (HQLAs)}}{\text{Total Nett Cash Outflows (NCOs) Over the Next 30 Calendar Days}} \geq 1. \quad (1)$$

The numerator of the LCR is the stock of HQLA. Under the standard, banks must hold a stock of unencumbered HQLA to cover the total net cash outflows over a 30-day period under the

prescribed stress scenario. In order to qualify as HQLA, assets should be liquid in markets during a time of stress and, in most cases, be eligible for use in central bank operations. Certain types of assets within HQLA are subject to a range of haircuts. HQLA are comprised of Level 1 assets (L1As) and Level 2 assets (L2As). L1As generally include cash, central bank reserves, and certain marketable securities backed by sovereigns and central banks, among others. These assets are typically of the highest quality and the most liquid, and there is no limit on the extent to which a bank can hold these assets to meet the LCR. L2As are comprised of Level 2A assets (L2AAs) and Level 2B assets (L2BAs). L2AAs include, for example, certain government securities, covered bonds and corporate debt securities. L2BAs include lower rated corporate bonds, residential mortgage backed securities and equities that meet certain conditions. L2As may not in aggregate account for more than 40 % of a bank's stock of HQLA. L2BAs may not account for more than 15 % of a bank's total stock of HQLA (see, for instance, [1] and [13]).

The denominator of the LCR is the total net cash outflows. It is defined as total expected cash outflows, minus total expected cash inflows, in the specified stress scenario for the subsequent 30 calendar days. Total expected cash outflows are calculated by multiplying the outstanding balances of various categories or types of liabilities and off-balance sheet commitments by the rates at which they are expected to run off or be drawn down. Total expected cash inflows are calculated by multiplying the outstanding balances of various categories of contractual receivables by the rates at which they are expected to flow in. Total cash inflows are subject to an aggregate cap of 75 % of total expected cash outflows, thereby ensuring a minimum level of HQLA holdings at all times (see, for instance, [11] and [13]).

The standard requires that, in the absence of financial stress, the value of the ratio be no lower than 100 % (i.e., the stock of HQLA should at least equal total net cash outflows). Banks are expected to meet this requirement on an ongoing basis and hold a stock of unencumbered HQLA as a defence against the potential onset of liquidity stress. During a period of financial stress, however, banks may use their stock of HQLA, thereby falling below 100 % (see, for instance, [11]).

## 1.1. Overview of the literature

Our contribution has strong connections with [7], [10], [12] and [13] that deals with subprime mortgage funding and liquidity risk and Basel III liquidity regulation, respectively.

The working paper [7] examines large capital injections by U.S. financial institutions from 2000 to 2009. These infusions include private as well as government cash injections under the Troubled Asset Relief Program (TARP). The sample period covers both business cycle expansions and contractions, and the recent financial crisis. Elyasiani, Mester and Pagano show that more financially constrained institutions were more likely to have raised capital through private market offerings during the period prior to TARP, and firms receiving a TARP injection tended to be riskier and more levered. In the case of TARP recipients, they appeared to finance an increase in lending (as a share of assets) with more stable financing sources such as core deposits, which lowered their liquidity risk. However, in [7], Elyasiani, Mester and Pagano find no evidence that banks' capital adequacy increased after the capital injections. In this book chapter, we regard the LCR as a measure of liquidity risk. We check the tendencies of this measure over the sample period 2002 to 2012 and make conclusions about it (see, also, [13]).

The technique employed in this book chapter is heavily reliant on the one used in [10]. In that paper, Mukuddem-Petersen and Petersen consider the application of stochastic optimization theory to asset and capital adequacy management in banking. The study is motivated by new banking regulation that emphasizes risk minimization practices associated with assets and regulatory capital. The analysis in [10] depends on the dynamics of the capital adequacy ratio (CAR), that we compute by dividing regulatory bank capital (RBC) by risk weighted assets (RWAs). Furthermore, Mukuddem-Petersen and Petersen demonstrate how the CAR can be optimized in terms of bank equity allocation and the rate at which additional debt and equity is raised. In either case, the dynamic programming algorithm for stochastic optimization is employed to verify the results. Also, in [10], Mukuddem-Petersen and Petersen provide an illustration of aspects of bank management practice in relation to this regulation. In the current chapter, the same technique is employed (see, also, [13]).

In [12], we use actuarial methods to solve a nonlinear stochastic optimal liquidity risk management problem with deposit inflow rates and marketable securities allocation as controls. The main objective in [12] is to minimize liquidity risk in the form of funding and credit crunch risk in an incomplete market. In order to accomplish this, we construct a stochastic model that incorporates mortgage and deposit reference processes. However, the current chapter is an improvement on [12] in that bank balance sheet features play a more prominent role (see Sections 2, 3 and 5 for more details).

In order to construct our LCR model, we take into account results obtained in [13] in a discrete-time framework (see, also, [8]). In the aforementioned book, we estimate the LCR and NSFR by applying approximation techniques to banking data from a cross section of countries. We find that these Basel III risk measures have low information values and are relatively poor indicators of liquidity risk. Our results, in [13], show that as the LCR increases (decreases) the probability of failure decreases (increases) for both Class I (internationally active banks with Tier 1 capital in excess of US \$ 4 billion) and II (the rest) banks. Our contribution is distinct from the aforementioned in the following respects. Firstly, our analysis has a heavy reliance on the derivation of a stochastic model for LCR dynamics that depends mainly on the liquidity provisioning rate, HQLA returns and NCO outflows. Secondly, we obtain an analytic solution of a particular type to our stochastic bank LCR problem (with a quadratic objective function) that we pose. Finally, the optimal choices for the cash injection and asset allocation are both expressed in terms of a LCR reference process. To our knowledge such processes have not been considered for LCRs before. The study is particularly significant because the Basel III LCR will be implemented on Thursday, 1 January 2015 on a global scale (see, for instance, [2], [3], [4] and [5]). In this book chapter, we extend the analysis in [13] to continuous time.

## 1.2. Outline of the book chapter

In short, this book chapter advances our knowledge of Basel III liquidity by investigating the LCR global liquidity standard (see, for instance, [6] and [14]) in an optimization context. In particular, in Section 2, a theoretical-quantitative model is constructed by considering the dynamics of the HQLAs and NCOs. Section 3 produces two parameters that are able to be controlled, viz., the liquidity provisioning rate and HQLA allocation. The main motivation for studying LCR dynamics is to show that, in principle, banks are able to control their liquidity via an appropriate provisioning strategy. This should ensure that the said ratio

does not move below an acceptable level. The control theoretic liquidity problem is to meet LCR targets with as little additional liquidity provisioning (essentially corresponding to cash injections in our chapter) as possible and optimal HQLA allocation. Section 5 provides conclusions and future directions. As was mentioned before, the additional provisioning may arise from an inflow of cash injections. We choose examples to illustrate that Basel III liquidity regulation resulted from both problematic liquidity structures and unexpected cash outflows (see Section 5 for more details).

## 2. A Liquidity Coverage Ratio Model

In this section, we model HQLAs, NCOs and LCR in a stochastic framework by following [10] very closely. This is important for solving the optimal LCR control problem outlined subsequently in Section 3.

### 2.1. Description of the Liquidity Coverage Ratio Model

Before the 2007-2009 financial crisis, banks were prosperous with high liquidity provisioning rates, low interest rates and soaring cash outflows. This was followed by the collapse of the housing market, exploding default rates and the effects thereafter. The LCR was developed to promote short-term resilience of a bank's liquidity risk profile. This standard aims to ensure that a bank has an adequate stock of unencumbered HQLAs that consist of cash or assets that can be converted into cash at little or no loss of value in private markets to meet its liquidity needs for a 30 calendar day liquidity stress scenario (see, for instance, [1], [3] and [4]). In order to make our analysis tractable, we make the following assumption about our LCR model.

**Assumption 2.1. (Filtered Probability Space and Time Index)** Assume that we have a filtered probability space  $(\Omega, \mathcal{F}, \mathbf{P})$  with filtration  $\{\mathcal{F}_t\}_{t \geq 0}$  on a time index set  $T = [t_0, t_1]$ .

Subsequently, we study a system of stochastic differential equations (SDEs) that value HQLAs at time  $t$  as  $x^1 : \Omega \times T \rightarrow \mathbb{R}^+$  (compare with [10]). Here, HQLAs,  $x_t^1$ , are stochastic because they are dependent on the stochastic rates of return on L1As and L2As (see [12] for more details). Also, NCOs at time  $t$ ,  $x_t^2$ , with  $x^2 : \Omega \times T \rightarrow \mathbb{R}^+$  are stochastic because their value has a reliance on random cash in- and outflows as well as liquidity provisioning. Furthermore, for  $x : \Omega \times T \rightarrow \mathbb{R}^2$  we use the notation  $x_t$  to denote

$$x_t = \begin{bmatrix} x_t^1 \\ x_t^2 \end{bmatrix}$$

and present the LCR,  $l : \Omega \times T \rightarrow \mathbb{R}^+$ , by

$$l_t = x_t^1 / x_t^2 = x_t^1 \cdot (x_t^2)^{-1}. \quad (2)$$

It is important for banks that  $l_t$  in (2) has to be sufficiently high to ensure high LCRs. In fact, as was mentioned before, Basel III sets the minimum value of the LCR at 1. Obviously, low values of  $l_t$  indicate that the bank has decreased liquidity and is at high risk of causing a credit crunch (see, for instance, [13]).

Bank liquidity has a heavy reliance on liquidity provisioning rates. This rate should be reduced for high LCRs and increased beyond the normal rate when bank LCRs are low. In the sequel, the stochastic process  $u^1 : \Omega \times T \rightarrow \mathbb{R}^+$  is the *normal rate of liquidity provisioning per monetary unit of the bank's NCOs* whose value at time  $t$  is denoted by  $u_t^1$ . In this case,  $u_t^1 dt$  is the normal liquidity provisioning rate per unit of the bank's NCOs over the time period  $(t, t + dt)$ . A related concept is the *adjustment to the rate of liquidity provisioning per monetary unit of the bank's NCOs for surplus or deficit*,  $u^2 : \Omega \times T \rightarrow \mathbb{R}^+$ , that depends on the LCR. In the case of liquidity deficit, during stress scenarios, this adjustment rate can correspond to a cash injection rate. Here the amount of surplus or deficit is reliant on the excess of HQLAs over NCOs. We denote the sum of  $u^1$  and  $u^2$  by the *liquidity provisioning rate*  $u^3 : \Omega \times T \rightarrow \mathbb{R}^+$ , i.e.,

$$u_t^3 = u_t^1 + u_t^2, \text{ for all } t. \quad (3)$$

The following assumption is made in order to model the LCR in a stochastic framework (compare with [10]).

**Assumption 2.2. (Liquidity Provisioning Rate)** *The liquidity provisioning rate,  $u^3$ , is predictable with respect to  $\{\mathcal{F}_t\}_{t \geq 0}$  and provides us with a means of controlling bank LCR dynamics (see (3) for more details).*

The closed loop system will be defined such that Assumption 2.2 is met, as we shall see in the sequel. In times of deficit, for (3), we should choose the cash injection rate,  $u^2$ , sufficiently large in order to guarantee bank liquidity. In reality, cash injections are subject to more stringent conditions (see, also, [13]).

Before and during the financial crisis, the LCR decreased significantly as extensive cash outflows took place with a consequent rising of NCOs. By contrast, banks predicted continued growth in the financial markets (see, for instance, [14]). The dynamics of the *outflows per monetary unit of the bank's NCOs*,  $e : \Omega \times T \rightarrow \mathbb{R}$ , is given by

$$de_t = r_t^e dt + \sigma_t^e dW_t^e, \quad e(t_0) = e_0, \quad (4)$$

where  $e_t$  is the outflows per NCO monetary unit,  $r^e : T \rightarrow \mathbb{R}$  is the rate of outflows per monetary unit of the bank's NCOs, the scalar  $\sigma^e : T \rightarrow \mathbb{R}$ , is the volatility in the outflows per NCO unit and  $W^e : \Omega \times T \rightarrow \mathbb{R}$  is standard Brownian motion (compare with [10]). Moreover, we consider

$$dh_t = r_t^h dt + \sigma_t^h dW_t^h, \quad h(t_0) = h_0, \quad (5)$$



where the stochastic processes  $h : \Omega \times T \rightarrow \mathbb{R}^+$  is the *investment return on bank HQLAs per monetary unit of HQLAs*,  $r^h \rightarrow \mathbb{R}^+$  is the rate of HQLA return per HQLA unit, the scalar  $\sigma^h : T \rightarrow \mathbb{R}$ , is the volatility in the rate of HQLA returns and  $W^h : \Omega \times T \rightarrow \mathbb{R}$  is standard Brownian motion. Before the 2007-2009 financial crisis, riskier HQLA returns were much higher than those of riskless reserves, making the former a more attractive but much riskier investment (see, also, [13]). During and after the crisis, this tendency reversed.

**Assumption 2.3. (HQLA Classes)** Suppose from the outset that bank HQLAs can be classified into  $n + 1$  asset classes. One of these HQLAs is risk free (like Central Bank reserves) while the HQLAs 1, 2, ...,  $n$  have some risk associated with them.

Riskier HQLAs evolve continuously in time and are modeled using a  $n$ -dimensional Brownian motion. In this multidimensional context, the *investment returns on bank HQLAs in the  $k$ -th HQLA per monetary unit of the  $k$ -th HQLA* is denoted by  $y_t^k$ ,  $k \in \mathbb{N}_n = \{0, 1, 2, \dots, n\}$  where  $y : \Omega \times T \rightarrow \mathbb{R}^{n+1}$ . Thus, the return per HQLA unit may be given by

$$y = (R(t), y_t^1, \dots, y_t^n),$$

where  $R(t)$  represents the return on reserves and  $y_t^1, \dots, y_t^n$  represent riskier HQLA returns. Furthermore, we can model  $y$  as

$$dy_t = r_t^y dt + \Sigma_t^y dW_t^y, \quad y(t_0) = y_0, \quad (6)$$

where  $r^y : T \rightarrow \mathbb{R}^{n+1}$  denotes the rate of asset returns,  $\Sigma_t^y \in \mathbb{R}^{(n+1) \times n}$  is a covariance matrix of HQLA returns and  $W^y : \Omega \times T \rightarrow \mathbb{R}^n$  is standard Brownian. We assume that the investment strategy  $\pi : T \rightarrow \mathbb{R}^{n+1}$  is outside the simplex

$$S = \{\pi \in \mathbb{R}^{n+1} : \pi = (\pi^0, \dots, \pi^n)^T, \pi^0 + \dots + \pi^n = 1, \pi^0 \geq 0, \dots, \pi^n \geq 0\}.$$

In this case, short selling is possible. The *investment return on bank HQLAs* is then  $h : \Omega \times R \rightarrow \mathbb{R}^+$ , where the dynamics of  $h$  can be written as

$$dh_t = \pi_t^T dy_t = \pi_t^T r_t^y dt + \pi_t^T \Sigma_t^y dW_t^y.$$

This notation can be simplified as follows. We denote

$$\begin{aligned}
 r^R(t) &= r^{y^0}(t), \quad r^R : T \rightarrow \mathbb{R}^+, \text{ the rate of return on riskless assets,} \\
 r_t^y &= (r^R(t), \tilde{r}_t^{y^T} + r^R(t)1_n)^T, \quad \tilde{r}^y : T \rightarrow \mathbb{R}^n, \\
 \pi_t &= (\pi_t^0, \tilde{\pi}_t^T)^T = (\pi_t^0, \pi_t^1, \dots, \pi_t^k)^T, \quad \tilde{\pi} : T \rightarrow \mathbb{R}^k, \\
 \Sigma_t^y &= \begin{pmatrix} 0 & \cdots & 0 \\ \tilde{\Sigma}_t^y \end{pmatrix}, \quad \tilde{\Sigma}_t^y \in \mathbb{R}^{n \times n}, \\
 \tilde{C}_t &= \tilde{\Sigma}_t^y \tilde{\Sigma}_t^{y^T}. \text{ Then, we have that} \\
 \pi_t^T r_t^y &= \pi_t^0 r^R(t) + \tilde{\pi}_t^{jT} \tilde{r}_t^y + \tilde{\pi}_t^{jT} r^R(t)1_n = r^R(t) + \tilde{\pi}_t^{T} \tilde{r}_t^y, \\
 \pi_t^T \Sigma_t^y dW_t^y &= \tilde{\pi}_t^T \tilde{\Sigma}_t^y dW_t^y, \\
 dh_t &= [r^R(t) + \tilde{\pi}_t^T \tilde{r}_t^y]dt + \tilde{\pi}_t^T \tilde{\Sigma}_t^y dW_t^y, \quad h(t_0) = h_0.
 \end{aligned} \tag{7}$$

Next, we take  $i : \Omega \times T \rightarrow \mathbb{R}^+$  as the *increase of NCOs before outflows per monetary unit of NCOs*,  $r^i : T \rightarrow \mathbb{R}^+$  is the rate of NCO increase before outflows per monetary unit of NCOs, the scalar  $\sigma^i \in \mathbb{R}$  is the volatility in the NCO increase before outflows per monetary unit of NCOs and  $W^i : \Omega \times T \rightarrow \mathbb{R}$  represents standard Brownian motion (compare with [10]). Then, we set

$$di_t = r_t^i dt + \sigma^i dW_t^i, \quad i(t_0) = i_0. \tag{8}$$

The stochastic process  $i_t$  in (8) may typically originate from NCOs that have recently taken place or instability in the value of pre-existing NCOs.

Next, we develop a simple stochastic model that replaces a more realistic system that emphasizes features that are specific to our particular study (see, also, [13]). In our situation, we derive models for HQLAs,  $x^1$ , and NCOs,  $x^2$ , given by

$$\begin{aligned}
 dx_t^1 &= x_t^1 dh_t + x_t^2 u_t^3 dt - x_t^2 de_t \\
 &= [r^R(t)x_t^1 + x_t^1 \tilde{\pi}_t^{jT} \tilde{r}_t^y + x_t^2 u_t^1 + x_t^2 u_t^2 - x_t^2 r_t^e]dt \\
 &\quad + [x_t^1 \tilde{\pi}_t^T \tilde{\Sigma}_t^y dW_t^y - x_t^2 \sigma^e dW_t^e],
 \end{aligned} \tag{9}$$

$$\begin{aligned}
 dx_t^2 &= x_t^2 di_t - x_t^2 de_t \\
 &= x_t^2 [r_t^i dt + \sigma^i dW_t^i] - x_t^2 [r_t^e dt + \sigma^e dW_t^e] \\
 &= x_t^2 [r_t^i - r_t^e]dt + x_t^2 [\sigma^i dW_t^i - \sigma^e dW_t^e].
 \end{aligned} \tag{10}$$

The SDEs (9) and (10) may be rewritten into matrix-vector form in the following way (compare with [10]).



**Definition 2.4. (Stochastic System for the LCR Model)** Define the stochastic system for the LCR model as

$$dx_t = A_t x_t dt + N(x_t) u_t dt + a_t dt + S(x_t, u_t) dW_t, \quad (11)$$

with the various terms in this SDE being

$$\begin{aligned} u_t &= \begin{bmatrix} u_t^2 \\ \tilde{\pi}_t \end{bmatrix}, \quad u : \Omega \times T \rightarrow \mathbb{R}^{n+1}, \\ A_t &= \begin{bmatrix} r^R(t) & -r_t^e \\ 0 & r_t^i - r_t^e \end{bmatrix}; \\ N(x_t) &= \begin{bmatrix} x_t^2 & x_t^1 \tilde{r}_t^{yT} \\ 0 & 0 \end{bmatrix}; \quad a_t = \begin{bmatrix} x_t^2 u_t^1 \\ 0 \end{bmatrix}; \\ S(x_t, u_t) &= \begin{bmatrix} x_t^1 \tilde{\pi}_t^T \tilde{\Sigma}_t^y & -x_t^2 \sigma^e & 0 \\ 0 & -x_t^2 \sigma^e & x_t^2 \sigma^i \end{bmatrix}; \\ W_t &= \begin{bmatrix} W_t^y \\ W_t^e \\ W_t^i \end{bmatrix}, \end{aligned}$$

where  $W_t^y$ ,  $W_t^e$  and  $W_t^i$  are mutually (stochastically) independent standard Brownian motions. It is assumed that for all  $t \in T$ ,  $\sigma_t^e > 0$ ,  $\sigma_t^i > 0$  and  $\tilde{C}_t > 0$  (compare with [10]).

We can rewrite (11) as follows.

$$\begin{aligned} N(x_t) u_t &:= \begin{bmatrix} x_t^2 \\ 0 \end{bmatrix} u_t^2 + \begin{bmatrix} x_t^1 \tilde{r}_t^{yT} \\ 0 \end{bmatrix} \tilde{\pi}_t \\ &:= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_t u_t^3 + \sum_{j=1}^n \begin{bmatrix} x_t^1 \tilde{r}_t^{y,j} \\ 0 \end{bmatrix} \tilde{\pi}_t^j \\ &:= B_0 x_t u_t^0 + \sum_{j=1}^n \begin{bmatrix} \tilde{r}_t^{y,j} & 0 \\ 0 & 0 \end{bmatrix} x_t \tilde{\pi}_t^j \\ &:= \sum_{j=0}^n [B^j x_t] u_t^j; \end{aligned}$$

and

$$\begin{aligned} S(x_t, u_t)dW_t &= \begin{bmatrix} [\tilde{\pi}_t^T \tilde{C}_t \tilde{\pi}_t]^{1/2} & 0 \\ 0 & 0 \end{bmatrix} x_t dW_t^1 + \\ &+ \begin{bmatrix} 0 & -\sigma^e \\ 0 & -\sigma^e \end{bmatrix} x_t dW_t^2 + \begin{bmatrix} 0 & 0 \\ 0 & \sigma^i \end{bmatrix} x_t dW_t^3 \\ &= \sum_{j=1}^3 [M^j(u_t)x_t] dW_t^j, \end{aligned}$$

where  $M^j(u_t)$  is the matrix notation used to denote matrices with entries related to  $u_t$ . Furthermore,  $W^1$ ,  $W^2$ , and  $W^3$  represent  $W^y$ ,  $W^e$ , and  $W^i$ , respectively. From (11) it is evident that  $u = (u^2, \tilde{\pi})$  affects only the SDE of  $x_t^1$  but not that of  $x_t^2$ . In particular, for (11) we have that  $\tilde{\pi}$  affects the variance of  $x_t^1$  and the drift of  $x_t^1$  via the term  $x_t^1 \tilde{r}_t^y \tilde{\pi}_t$ . On the other hand,  $u^2$  affects only the drift of  $x_t^1$ . Then (11) becomes

$$dx_t = A_t x_t dt + \sum_{j=0}^n [B^j x_t] u_t^j dt + a_t dt + \sum_{j=1}^3 [M^j(u_t)x_t] dW_t^j. \quad (12)$$

## 2.2. Description of the simplified LCR model

The model can be simplified if attention is restricted to the system with the LCR, as stated earlier, denoted in this section by  $l_t = x_t^1 \cdot (x_t^2)^{-1}$  (compare with [10]).

**Definition 2.5. (Stochastic Model for a Simplified LCR)** Define the simplified LCR system by the SDE

$$\begin{aligned} dx_t &= x_t [r^R(t) + r_t^e - r_t^i + (\sigma^e)^2 + (\sigma^i)^2 + \tilde{r}_t^y \tilde{\pi}_t] dt \\ &+ [u_t^1 + u_t^2 - r_t^e - (\sigma^e)^2] dt \\ &+ [(\sigma^e)^2 (1 - x_t)^2 + (\sigma^i)^2 x_t^2 + x_t^2 \tilde{\pi}_t^T \tilde{C}_t \tilde{\pi}_t]^{1/2} d\bar{W}_t, \quad x(t_0) = x_0. \end{aligned} \quad (13)$$

The model is derived as follows. The starting point is the two-dimensional SDE for  $x = (x^1, x^2)^T$  as in the equations (9) and (10). Next, we use the Itô's formula (see, for instance, [15]) to determine

$$\begin{aligned}
d(x_t^2)^{-1} &= -(x_t^2)^{-2}dx_t^2 + \frac{1}{2}2(x_t^2)^{-3}d < x^2, x^2 > t \\
&= [-(x_t^2)^{-1}(r_t^i - r_t^e) + (x_t^2)^{-1}((\sigma^e)^2 + (\sigma^i)^2)]dt \\
&\quad - (x_t^2)^{-1} [0 - \sigma^e \sigma^i] dW_t, \\
dx_t &= x_t^1 d(x_t^2)^{-1} + (x_t^2)^{-1} dx_t^1 + d < x^1, (x^2)^{-1} > t \\
&= [r^R(t)x_t - r_t^e + u_t^1 + u_t^2 + x_t \tilde{r}_t^y \tilde{\pi}_t \\
&\quad - x_t[r_t^i - r_t^e] + x_t((\sigma^e)^2 + (\sigma^i)^2) - (\sigma^e)^2]dt \\
&\quad + (x_t \tilde{\pi}_t^T \tilde{\Sigma}_t^y - \sigma^e(1 - x_t) - \sigma^i x_t) dW_t \\
&= x_t[r^R(t) + r_t^e - r_t^i + (\sigma^e)^2 + (\sigma^i)^2 + \tilde{r}_t^y \tilde{\pi}_t]dt + \\
&\quad + [u_t^1 + u_t^2 - r_t^e - (\sigma^e)^2]dt \\
&\quad + \left[ (\sigma^e)^2(1 - x_t)^2 + (\sigma^i)^2(x_t)^2 + (x_t)^2 \tilde{\pi}_t^T \tilde{C}_t \tilde{\pi}_t \right]^{1/2} d\bar{W}_t,
\end{aligned}$$

for stochastic  $\bar{W} : \Omega \times T \rightarrow \mathbb{R}$  that is a standard Brownian motion. Note that in the drift of the SDE (13), the term

$$-r_t^e + x_t r_t^e = -r_t^e(x_t - 1),$$

appears because it models the effect of depreciation of both HQLAs and NCOs. Similarly, the term  $-(\sigma^e)^2 + x_t(\sigma^e)^2 = (\sigma^e)^2(x_t - 1)$  appears.

The predictions made by our previously constructed model are consistent with the empirical evidence in contributions such as [13]. For instance, in much the same way as we do, [13] describes how NCOs affect LCRs. On the other hand, to the best of our knowledge, the modeling related to collateral and LCR reference processes (see Section 3 for a comprehensive discussion) have not been tested in the literature before.

### 3. Optimal Basel III liquidity coverage ratios

In order to determine an optimal cash injection rate (seen as an adjustment to the normal provisioning rate) and HQLA allocation strategy, it is imperative that a well-defined objective function with appropriate constraints is considered. The choice has to be carefully made in order to avoid ambiguous solutions to our stochastic control problem (compare with [10]).

#### 3.1. The optimal bank LCR problem

As in [10], in our contribution, we choose to determine a control law  $g(t, x_t)$  that minimizes the cost function  $J : \mathcal{G}_A \rightarrow \mathbb{R}^+$ , where  $\mathcal{G}_A$  is the class of admissible control laws

$$\mathcal{G}_A = \{g : T \times \mathcal{X} \rightarrow \mathcal{U} | g \text{ Borel measurable} \text{ and there exists an unique solution to the closed-loop system}\}, \quad (14)$$

with the closed-loop system for  $g \in \mathcal{G}_A$  being given by

$$\begin{aligned} dx_t = & A_t x_t dt + \sum_{j=0}^n B^j x_t g^j(t, x_t) dt + a_t dt \\ & + \sum_{j=1}^3 M^j(g(t, x_t)) x_t dW_t^j, \quad x(t_0) = x_0. \end{aligned} \quad (15)$$

Furthermore, the cost function,  $J : \mathcal{G}_A \rightarrow \mathbb{R}^+$ , of the LCR problem is given by

$$\begin{aligned} J(g) = & \mathbb{E} \left[ \int_{t_0}^{t_1} \exp(-r^f(l - t_0)) b(l, x_l, g(l, x_l)) dl \right. \\ & \left. + \exp(-r^f(t_1 - t_0)) b^1(x(t_1)) \right], \end{aligned} \quad (16)$$

where  $g \in \mathcal{G}_A$ ,  $T = [t_0, t_1]$  and  $b^1 : \mathcal{X} \rightarrow \mathbb{R}^+$  is a Borel measurable function (compare with [10]). Furthermore,  $b : T \times \mathcal{X} \times \mathcal{U} \rightarrow \mathbb{R}^+$  is formulated as

$$b(t, x, u) = b^2(u^2) + b^3(x^1/x^2),$$

for  $b^2 : \mathcal{U}_2 \rightarrow \mathbb{R}^+$  and  $b^3 : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ . Also,  $r^f \in \mathbb{R}$  is called the *NCO forecasting rate*, where  $b^1$ ,  $b^2$  and  $b^3$  are chosen below. In order to clarify the stochastic problem, the following assumption is made.

**Assumption 3.1. (Admissible Class of Control Laws)** Assume that  $\mathcal{G}_A \neq \emptyset$ .

We are now in a position to state the stochastic optimal control problem for a continuous-time LCR model that we solve (compare with [10]). The said problem may be formulated as follows.

**Problem 3.2. (Optimal Bank LCR Problem)** Consider the stochastic system (15) for the LCR problem with the admissible class of control laws,  $\mathcal{G}_A$ , given by (14) and the cost function,  $J : \mathcal{G}_A \rightarrow \mathbb{R}^+$ , given by (16). Solve

$$\inf_{g \in \mathcal{G}_A} J(g),$$

that amounts to determining the value  $J^*$ , given by

$$J^* = \inf_{g \in \mathcal{G}_A} J(g),$$

and the optimal control law  $g^*$ , if it exists,

$$g^* = \arg \min_{g \in \mathcal{G}_A} J(g) \in \mathcal{G}_A.$$

### 3.2. Optimal bank LCRs in the simplified case

In this section, we determine a solution to Problem 3.2 in the case where the term  $[t_0, t_1]$  is fixed. In order to find the optimal control processes, we use the dynamic programming algorithm for stochastic optimization where we consider an appropriate Hamilton-Jacobi-Bellman equation (HJBE). In the sequel, we assume that the optimal control laws exist, with the objective function,  $J$ , given by (16) being continuous twice-differentiable. Then a combination of integral calculus and Itô's formula (see, for instance, [15]) shows that the value function  $v$  satisfies (20) and (21).

Consider the simplified system (13) for the LCR problem with the admissible class of control laws,  $\mathcal{G}_A$ , given by (14) but with  $\mathcal{X} = \mathbb{R}$  (compare with [10]). In this section, we have to solve

$$\begin{aligned} & \inf_{g \in \mathcal{G}_A} J(g), \\ J^* &= \inf_{g \in \mathcal{G}_A} J(g), \end{aligned} \tag{17}$$

$$\begin{aligned} J(g) &= \mathbb{E} \left[ \int_{t_0}^{t_1} \exp(-r^f(l - t_0)) [b^2(u_t^2) + b^3(x_t)] dt \right. \\ & \quad \left. + \exp(-r^f(t_1 - t_0)) b^1(x(t_1)) \right], \end{aligned} \tag{18}$$

where  $b^1 : \mathbb{R} \rightarrow \mathbb{R}^+$ ,  $b^2 : \mathbb{R} \rightarrow \mathbb{R}^+$  and  $b^3 : \mathbb{R}^+ \rightarrow \mathbb{R}^+$  are all Borel measurable functions. For the simplified case, the optimal cost function (17) is determined with the simplified cost function,  $J(g)$ , given by (18). In this case, assumptions have to be made in order to find a solution for the optimal cost function,  $J^*$  (compare with [10]). Next, we state an important result about optimal bank coverage ratios in the simplified case.

#### Theorem 3.3. (Optimal Bank LCRs in the Simplified Case)

Suppose that  $g^{2*}$  and  $g^{3*}$  are the components of the optimal control law,  $g^*$ , that deal with the optimal cash injection rate,  $u^{2*}$ , and optimal HQLA allocation,  $\pi^{k*}$ , respectively. Consider the nonlinear optimal stochastic control problem for the simplified LCR system (13) formulated in Problem 3.2. Suppose that the following assumptions hold.

1. The cost function is assumed to satisfy

$$\begin{aligned} b^2(u^2) &\in C^2(\mathbb{R}), \\ \lim_{u^2 \rightarrow -\infty} D_{u^2} b^2(u^2) &= -\infty, \quad \lim_{u^2 \rightarrow +\infty} D_{u^2} b^2(u^2) = +\infty; \\ D_{u^2 u^2} b^2(u^2) &> 0, \quad \forall u^2 \in \mathbb{R}, \end{aligned} \quad (19)$$

with the differential operator,  $D$ , that is applied in this case to function  $b^2$ .

2. There exists a function  $v : T \times \mathbb{R} \rightarrow \mathbb{R}$ ,  $v \in C^{1,2}(T \times \mathcal{X})$ , that is a solution of the HJBE given by

$$\begin{aligned} 0 = & D_t v(t, x) + \frac{1}{2} [(\sigma^e)^2 (1-x)^2 + (\sigma^i)^2 (x_t)^2] D_{xx} v(t, x) \\ & + x(r^R(t) + r_t^e - r_t^i + (\sigma^e)^2 + (\sigma^i)^2) D_x v(t, x) \\ & + [u_t^1 - r_t^e - (\sigma^e)^2] D_x v(t, x) \\ & + u_t^{2*} D_x v(t, x) + \exp(-r^f(t - t_0)) b^2(u_t^{2*}) \\ & + \exp(-r^f(t - t_0)) b^3(x) - \frac{[D_x v(t, x)]^2}{2 D_{xx} v(t, x)} \tilde{r}_t^y \tilde{C}_t^{-1} \tilde{r}_t^y, \end{aligned} \quad (20)$$

$$v(t_1, x) = \exp(-r^f(t_1 - t_0)) b^1(x), \quad (21)$$

where  $u^{2*}$  is the unique solution of the equation

$$0 = D_x v(t, x) + \exp(-r^f(t - t_0)) D_{u^2} b^2(u_t^2). \quad (22)$$

Then the optimal control law is

$$g^{2*}(t, x) = u^{2*}, \quad g^{2*} : T \times \mathcal{X} \rightarrow \mathbb{R}^+, \quad (23)$$

with  $u^{2*} \in \mathcal{U}_2$  the unique solution of the equation (22)

$$\tilde{\pi}^* = -\frac{D_x v(t, x)}{x D_{xx} v(t, x)} \tilde{C}_t^{-1} \tilde{r}_t^y, \quad (24)$$

$$g^{3,k*}(t, x) = \min\{1, \max\{0, \tilde{\pi}^{k*}\}\}, \quad g^{3,k*} : T \times \mathcal{X} \rightarrow \mathbb{R}, \quad (25)$$

Furthermore, the value of the problem is

$$J^* = J(g^*) = \mathbb{E}[v(t, x_0)]. \quad (26)$$



Next, we choose a particular cost functions for which an analytic solution can be obtained for the value function and control laws (compare with [10]). The following theorem provides the optimal control laws for quadratic cost functions.

**Theorem 3.4. (Optimal Bank LCRs with Quadratic Cost Functions):** Consider the nonlinear optimal stochastic control problem for the simplified LCR system (13) formulated in Problem 3.2. Consider the cost function

$$J(g) = \mathbb{E} \left[ \int_{t_0}^{t_1} \exp(-r^f(l - t_0)) \left[ \frac{1}{2} c^2 ((u^2)^2(l)) + \frac{1}{2} c^3 (x_t - l^r)^2 \right] dl + \frac{1}{2} c^1 (x(t_1) - l^r)^2 \exp(-r^f(t_1 - t_0)) \right]. \quad (27)$$

We assume that the cost functions satisfy

$$b^1(x) = \frac{1}{2} c^1 (x - l^r)^2, \quad c^1 \in (0, \infty);$$

$$b^2(u^2) = \frac{1}{2} c^2 (u^2)^2, \quad c^2 \in (0, \infty); \quad (28)$$

$$b^3(x) = \frac{1}{2} c^3 (x - l^r)^2, \quad c^3 \in (0, \infty), \quad (29)$$

$l^r \in \mathbb{R}$ , called the reference value of the LCR

Define the first-order ODE

$$-\dot{q}_t = -(q_t)^2 / c^2 + c^3 + q_t 2(r^R(t) + r_t^e - r_t^i + (\sigma^e)^2 + (\sigma^i)^2) + q_t [-r^f - \tilde{r}_t^y \tilde{C}_t^{-1} \tilde{r}_t^y + (\sigma^e)^2 + (\sigma^i)^2], \quad q_{t_1} = c^1; \quad (30)$$

$$-\dot{x}_t^r = -c^3 (x_t^r - l^r) / q_t - x_t^r [r^R(t) + r_t^e - r_t^i + (\sigma^e)^2 + (\sigma^i)^2] - [u_t^1 - r_t^e - (\sigma^e)^2] - (x_t^r - 1)((\sigma^e)^2 + (\sigma^i)^2) - (\sigma^i)^2, \quad x^r(t_1) = l^r; \quad (31)$$

$$-\dot{l}_t = -r^f l_t + c^3 (x_t^r - l^r)^2 - q_t (\sigma^e)^2 (x_t^r - 1)^2 - q_t (\sigma^i)^2 (x_t^r)^2, \quad l(t_1) = 0. \quad (32)$$

The function  $x^r : T \rightarrow \mathbb{R}$  will be called the LCR reference (process) function. Then we have that the following hold.

- (a) There exist solutions to the ordinary differential equations (30), (31) and (32). Moreover, for all  $t \in T$ ,  $q_t > 0$ .

(b) The optimal control laws are

$$u_t^{2*} = -(x - x_t^r)q_t/c^2, \quad (33)$$

$$g^{2*}(t, x) = u^{2*}, \quad g^{2*} : T \times \mathcal{X} \rightarrow \mathbb{R}^+,$$

$$\tilde{\pi}_t^* = -\frac{(x - x_t^r)}{x} \tilde{C}_t^{-1} \tilde{r}_t^y, \quad g^{3*} : T \times \mathcal{X} \rightarrow \mathbb{R}^k, \quad (34)$$

$$g^{3,k*}(t, x) = \begin{cases} \tilde{\pi}^{k*}, & \text{if } \tilde{\pi}^{k*} \in [0, 1], \\ \min\{1, \max\{0, \tilde{\pi}_t^{k*}\}\}, & \text{otherwise,} \end{cases} \quad \forall k \in \mathbb{Z}^n. \quad (35)$$

(c) The value function and the value of the problem are

$$v(t, x) = \exp(-r^f(t_1 - t_0)) \left[ \frac{1}{2}(x - x_t^r)^2 q_t + \frac{1}{2} l_t \right], \quad (36)$$

$$J^* = J(g^*) = \mathbf{E}[v(t_0, x_0)]. \quad (37)$$

For the cost on the cash injection, the function (28) is considered, where the input variable  $u^2$  is restricted to the set  $\mathbb{R}^+$ . If  $u^2 > 0$  then the banks should acquire additional HQLAs. The cost function should be such that cash injections are maximized, hence  $u^2 > 0$  should imply that  $b^2(u^2) > 0$ . For Theorem 3.4 we have selected the cost function  $b^2(u^2) = \frac{1}{2}c^2(u^2)^2$ , given by (28). Here, both positive and negative values of  $u^2$  are penalized equally. An important reason for this is that an analytic solution of the value function can be determined (compare with [10]).

The reference process,  $l^r$ , may be 1 that is the threshold for the LCR standard. The cost on meeting liquidity provisioning will be encoded in a cost on the LCR. If the LCR,  $x > l^r$ , is strictly larger than a set value  $l^r$ , then there should be a strictly negative cost. If, on the other hand,  $x < l^r$ , then there may be a positive cost. We have selected the cost function  $b^3(x) = \frac{1}{2}c^3(x - l^r)^2$  in Theorem 3.4 given by (29). This is also done to obtain an analytic solution of the value function and that case by itself is interesting (see, for instance, [10] for more details). Another cost function that we can consider is

$$b^3(x) = c^3[\exp(x - l^r) + (l^r - x) - 1],$$

that is strictly convex and asymmetric in  $x$  with respect to the value  $l^r$ . For this cost function, it is reasonable that costs with  $x > l^r$  are penalized lower than those with  $x < l^r$ . Another cost function considered is to keep  $b^3(x) = 0$  for  $x < l^r$  (see, for instance, [10]).

#### 4. Numerical results for LCRs

In this section, we provide numerical-quantitative results about LCRs and their connections with HQLAs and NCOs to supplement the theoretical-quantitative treatment in Sections 2 and 3 (see [13] for more details). More precisely, we describe the LCR data and descriptive statistics for Class I and II banks for the sample period 2002 to 2012.

## 4.1. Description of banking data

In this subsection, we describe the banking data pertaining to LCRs.

### 4.1.1. Class I and II banks

We investigate liquidity for Class I banks that hold more than US \$ 4 billion in Tier 1 capital (T1K) and are internationally active. Moreover, we consider Class II banks that violate one or both of these conditions (see, for instance, [9] and [17]). In reality, some Class II banks considered could have been classified as Class I if they were internationally active. Nevertheless, these banks make a large contribution to the total assets of Class II banks. Invariably, all Class I banks can also be classified as large in that their gross total assets (GTA) exceed US \$ 3 billion. Many of the banks in our study come from jurisdictions affiliated to the BCBS and Macro-Economic Assessment Group (MAG).

Our investigation includes 157 Class I and 234 Class II LIBOR-based banks from 38 countries. These banks (with the number of Class I and Class II banks in parenthesis for each jurisdiction, as well as \* and ' denoting BCBS and MAG members, respectively) are located in Argentina\* (1,3), Australia\*' (5,2), Austria (2,5), Belgium\* (1,2), Botswana (1,1), Brazil\*' (3,1), Canada\*' (7,3), China\*' (7,1), Czech Republic (4,3), Finland (0,14), France\*' (5,5), Germany\*' (7,24), Hong Kong SAR\* (1,8), Hungary (1,2), India\* (6,6), Indonesia\* (1,3), Ireland (3,1), Italy\*' (2,11), Japan\*' (14,5), Korea\*' (6,4), Luxembourg\* (0,1), Malta (0,3), Mexico\*' (1,8), Namibia (0,1), the Netherlands\*' (3,13), Norway (1,6), Poland (0,5), Portugal (3,3), Russia\* (0,3), Saudi Arabia\* (4,1), Singapore\* (5,0), South Africa\* (4,5), Spain\*' (2,4), Sweden\* (4,0), Switzerland\*' (3,5), Turkey\* (7,1), United Kingdom\*' (8,5) and United States\*' (35,66). In order to limit depositor losses, all 38 jurisdictions have explicit deposit insurance schemes or implicit government protection schemes for banks.

### 4.1.2. Banking data restrictions

In our study, we did not consider Central Banks, subsidiaries, banks with incomplete (inconsistent or non-continuous) information nor observations with negative HQLA, NCO, ASF, RSF or other values (see, for instance, [9] and [17]). Furthermore, we use non-permanent samples that do not suffer from survivorship bias to study cross sectional patterns. For our sample, bank failure data for the period 2002 to 2012 was obtained from deposit insurance schemes or implicit government protection schemes. For instance, for the US, such data was obtained from the Federal Deposit Insurance Corporation (see [9] and [17] for more details). We choose the period 2002-2012 because available EMERG global liquidity data does not allow us to reliably determine the LCR and NSFR prior to 2002 (see, for instance, [9] and [17]).

### 4.1.3. Banking data computations

Estimating the LCR and NSFR using available EMERG public data proved to be a challenge. Firstly, the prescripts for these risk standards are sometimes ambiguous and subject to frequent regulatory amendment. For instance, the final rules relating to the LCR were only published on Monday, 7 January 2013.

Secondly, the EMERG global banking data has several limitations in terms of granularity and format when compared with the information required to determine the Basel III liquidity standards (see, for instance, [9] and [17]). In all instances, we had to make difficult choices when applying Basel III guidelines to such a large diversity of banks.

In the absence of suitable data, we were heavily dependent on the interpolation and extrapolation techniques discussed below. Firstly, it is clear that the LCR calculation requires information about liabilities with a remaining maturity of less than 1 month. However, quarterly EMERG data provides information about liabilities with a remaining maturity of less than 3 months. So we had to extrapolate the liabilities with a remaining maturity of 1 month. There are two approaches to doing this. In the first instance, we can assume the maturity schedule is evenly distributed, such that the amount of liabilities with a remaining maturity of less than 1 month equals 1/3 of the amount of liabilities with a remaining maturity of less than 3 months. This is the approach adopted in this chapter. Secondly, as a robustness check, we can assume an extreme case, such that all liabilities with a remaining maturity within 3 months mature within the first month. In this instance, the guidelines require dividing liabilities into subcategories of retail deposits, unsecured wholesale funding and secured funding with different run-off rates (see, for instance, [9] and [17]). However, the information available from the EMERG global data lacks such granularity. Out of necessity, we have to make assumptions on the distribution of subcategories within their primary category. Without additional information, we generally assume equal distribution of subcategories within the primary category. Finally, except for unused commitments, letters of credit and the net fair value of derivatives, we do not have the information required for calculating the liquidity needs of all other OBS items, such as increased liquidity needs related to downgrade triggers embedded in financing transactions, derivatives and other contracts. Therefore, our calculations of the LCR and NSFR are partial measures that capture a bank's liquidity risk as mainly reflected by its BS and to a lesser extent its OBS items (see [9] and [17] for more information).

## 4.2. 2002 to 2012 LCRs for class I and II banks

In this subsection, we provide 2002 to 2012 LCRs for Class I and II banks.

Table 1 shows that the LCR has been in a downward trend from 2002 through 2007. The average LCR had risen sharply from 2007 to 2009 and peaked in 2009. The general impression from Figure 1 is that the LCR time series is non-stationary.

## 4.3. Descriptive statistics for LCRs of class I and II banks

In this subsection, we provide 2002 to 2012 LCR descriptive statistics for Class I and II banks.

Table 2 reports the summary statistics of the approximate measures of the LCR for Class I banks, where the mean for the LCR is 74.96 %. In this table, the LCR displays positive skewness. The value of the kurtosis for the LCR in Table 2 is equal to or less than 3, that means that the distribution is flat. The LCR risk measure exhibits normality because the *p*-values are greater than 5 %. Nevertheless, the normality test is very sensitive to the number of observations and may only produce desirable and efficient results if observations are large. From Table 2, it is clear that, in the absence of empirical evidence, it is hard to conclude that the Basel III LCR standard had complied with these standards.

Bank Class			Bank Class		
Quarter	Class I	Class II	Quarter	Class I	Class II
02Q1	0.81832	0.84588	02Q2	0.79516	0.82194
02Q3	0.8264	0.85386	02Q4	0.86464	0.89376
03Q1	0.78884	0.77406	03Q2	0.74112	0.76608
03Q3	0.7334	0.7581	03Q4	0.71024	0.73416
04Q1	0.6562	0.6783	04Q2	0.62532	0.64638
04Q3	0.63304	0.65436	04Q4	0.6176	0.6384
05Q1	0.62532	0.64638	05Q2	0.6176	0.6384
05Q3	0.60988	0.63042	05Q4	0.6176	0.6384
06Q1	0.59444	0.61446	06Q2	0.57128	0.59052
06Q3	0.57128	0.59052	06Q4	0.54812	0.56658
07Q1	0.5404	0.5586	07Q2	0.579	0.5985
07Q3	0.55584	0.51456	07Q4	0.54812	0.56658
08Q1	0.72568	0.75012	08Q2	0.74884	0.77406
08Q3	0.74884	0.77406	08Q4	0.74412	0.76608
09Q1	0.88008	0.90972	09Q2	0.95728	1.08114
09Q3	1.02676	1.06134	09Q4	0.91566	0.99753
10Q1	0.93412	0.96558	10Q2	0.95376	0.96184
10Q3	0.96552	0.92568	10Q4	0.97868	0.94962
11Q1	0.98324	0.93366	11Q2	0.99288	0.91566
11Q3	0.99692	0.98578	11Q4	1.02391	1.08113
12Q1	1.07414	1.16982	12Q2	1.10664	1.19376
12Q3	1.16849	1.23778	12Q4	1.19281	1.29002

**Table 1.** 2002 to 2012 LCRs for Class I and II Banks

Descriptive Statistics of LCR for Class I and II Banks		
Parameter	Class I LCR	Class II LCR
Mean	0.748720	0.773430
Median	0.748840	0.774060
Maximum	1.026760	1.061340
Minimum	0.540400	0.514560
Std. Dev.	0.027670	0.143466
Skewness	0.027670	-0.02945
Kurtosis	1.825270	1.855696
Jarque-Bera	2.535599	2.406985
Probability	0.281450	0.300144
Sum	32.94368	34.03091
Sum Sq. Dev.	0.794993	0.885043
Observations	44	44

**Table 2.** Descriptive Statistics of LCR for Class I and II Banks

## 5. Conclusions and future directions

In this section, we draw conclusions about the LCR modeling and optimization and related numerical examples. Furthermore, we suggest possible topics for future research.

### 5.1. Conclusions

In this subsection, we make conclusions about the LCR model, optimal Basel III LCRs and numerical results for LCRs.

#### 5.1.1. Conclusions about the LCR model

One of the main contributions of this book chapter is the way the LCR dynamics model is constructed by using stochastic techniques. This model depends on HQLAs, NCOs as well as the liquidity provisioning rate. We believe that this is an addition to pre-existing literature because it captures some of the uncertainty associated with LCR variables. In this regard, we provide a theoretical-quantitative modeling framework for establishing bank LCR reference processes and the making of decisions about liquidity provisioning rates and asset allocation.

In Subsection 2.1, we mention the possibility of adjusting the cash injection rate depending on whether the bank is experiencing deficit or surplus liquidity. The latter occurs where cashflows into the banking system persistently exceed withdrawals of liquidity from the market by the central bank. This is reflected in holdings of reserves in excess of the central bank's required reserves. Transitional economies, for example, often attract large capital inflows as the economy opens and undergoes privatization. The effect of these inflows on liquidity is often magnified by central bank intervention in the foreign exchange market when there is upward pressure on the domestic currency. In the wartime economy, consumption is restricted and large amounts of involuntary savings accumulate until goods and services eventually become more widely available. Soviet-style economies have displayed widespread shortages and administered prices. This creates a situation of repressed inflation, whereby prices are too low relative to the money stock, leaving individuals with excess real balances. The importance of surplus liquidity for central banks is threefold and lies in its potential to influence: (1) the transmission mechanism of monetary policy; (2) the conduct of central bank intervention in the money market, and (3) the central bank's balance sheet and income.

#### 5.1.2. Conclusions about optimal basel III LCRs

We obtained an analytic solution to an optimal bank LCR problem with a quadratic objective function. In principle, this solution can assist in managing LCRs. Here, liquidity provisioning and HQLA allocation are expressed in terms of a reference process. To our knowledge such processes have not been considered for LCRs before. This chapter makes a clear connection between liquidity and financial crises in a numerical-quantitative framework.

An interpretation of the control laws given by (33) and (34) follows. In times of deficit, the cash injection rate,  $u^{2*}$ , is proportional to the difference between the LCR,  $x$ , and the reference process for this ratio,  $x^r$ . The proportionality factor is  $q_t/c^2$  that depends on the relative ratio of the cost function on  $u^2$  and the deviation from the reference ratio,  $(x - x^r)$ . The property that the control law is symmetric in  $x$  with respect to the reference process  $x^r$  is a direct consequence of the cost function  $b^r(x) = \frac{1}{2}c^3(x - x^r)^2$  being symmetric



with respect to  $(x - x^r)$ . The optimal portfolio distribution is proportional to the relative difference between the LCR and its reference process,  $(x - x_t^r)/x$ . This seems natural. The proportionality factor is  $\tilde{C}_t^{-1} \tilde{r}_t^y$  that represents the relative rates of asset return multiplied with the inverse of the corresponding variances. It is surprising that the control law has this structure. Apparently the optimal control law is not to liquidate first all HQLAs with the highest liquidity provisioning rate, then the HQLAs with the next to highest liquidity provisioning rate, etc. The proportion of all HQLAs depend on the relative weighting in  $\tilde{C}_t^{-1} \tilde{r}_t^y$  and not on the deviation  $(x - x_t^r)$ .

The novel structure of the optimal control law is the *LCR reference process*,  $x^r : T \rightarrow \mathbb{R}$ . The differential equation for this reference function is given by (31). This equation is new for the area of LCR control and therefore deserves discussion. The differential equation has several terms on its right-hand side that will be discussed separately. Consider the term

$$u_t^1 - r_t^e - (\sigma^e)^2.$$

This represents the difference between normal rate of liquidity provisioning per monetary unit of the bank's NCOs and NCO outflows, where  $r_t^e$  is the rate of outflow per monetary unit of NCOs. Note that if  $[u_t^1 - r_t^e - (\sigma^e)^2] > 0$ , then the reference LCR function can be increasing in time due to this inequality so that, for  $t > t_1$ ,  $x_t < l^r$ . The term  $c^3(x_t^r - l^r)/q_t$  models that if the reference LCR function is smaller than  $l^r$ , then the function has to increase with time. The quotient  $c^3/q_t$  is a weighting term that accounts for the running costs and for the effect of the solution of the Riccati differential equation. The term

$$x_t^r [r^R(t) + r_t^e - r_t^i + (\sigma^e)^2 + (\sigma^i)^2],$$

accounts for two effects. The difference  $r_t^e - r_t^i$  is the nett effect of the rate of outflows per monetary unit of the bank's NCOs,  $r_t^e$ , and rate of NCO increase before outflows per monetary unit of NCOs,  $r_t^i$ . The term  $r^R(t) + (\sigma^e)^2 + (\sigma^i)^2$  is the effect of NCO increase due to the reserves and the variance of riskier liquidity provisioning. The last term

$$(x_t^r - 1)((\sigma^e)^2 + (\sigma^i)^2) - (\sigma^i)^2,$$

accounts for the effect on HQLAs and NCOs. More information is obtained by streamlining the ODE for  $x^r$ . In order to accomplish this it is necessary to assume the following.

**Assumption 5.1. (Liquidity Parameters):** Assume that the parameters of the problem are all time-invariant and also that  $q$  has become constant with value  $q^0$ .

Then the differential equation for  $x^r$  can be rewritten as

$$\begin{aligned} -\dot{x}_t^r &= -k(x_t^r - m), \quad x^r(t_1) = l^r; \\ k &= (r^R + r^e - r^i + 2((\sigma^e)^2 + (\sigma^i)^2)) + c^3/q^0; \\ m &= \frac{l^r c^3/q^0 - (u^1 - r^e - (\sigma^e)^2) + (\sigma^e)^2}{(r^R + r^e - r^i + 2((\sigma^e)^2 + (\sigma^i)^2)) + c^3/q^0}. \end{aligned}$$

Because the finite horizon is an artificial phenomenon to make the optimal stochastic control problem tractable, it is of interest to consider the long term behavior of the LCR reference trajectory,  $x^r$ . If the values of the parameters are such that  $k > 0$  then the differential equation with the terminal condition is stable. If this condition holds then  $\lim_{t \downarrow 0} q_t = q^0$  and  $\lim_{t \downarrow 0} x_t^r = m$  where the down arrow prescribes to start at  $t_1$  and to let  $t$  decrease to 0. Depending on the value of  $m$ , the control law for at a time very far away from the terminal time becomes then,

$$\begin{aligned} u_t^{2*} &= -(x_t - m)q^0/c^2 = \begin{cases} > 0, & \text{if } x_t < m, \\ < 0, & \text{if } x_t > m, \end{cases} \\ \pi_t^* &= -\frac{(x_t - m)}{x_t} \tilde{C}r^j = \begin{cases} > 0, & \text{if } x_t < m, \\ < 0, & \text{if } x_t > m, \end{cases} \text{ if } \pi^* < 0 \text{ then set } \pi^* = 0, \end{aligned}$$

The interpretation for the two cases follows below.

Case 1 ( $x_t > m$ ): Then the LCR  $x$  is too high. This is penalized by the cost function hence the control law prescribes not to invest in riskier HQLAs. The payback advice is due to the quadratic cost function that was selected to make the solution analytically tractable. An increase in liquidity provisioning will increase NCOs that, in turn, will lower the LCR.

Case 2 ( $x_t < m$ ): The LCR  $x$  is too low. The cost function penalizes and the control law prescribes to invest more in riskier HQLAs. In this case, more funds will be available and credit risk on the balance sheet will decrease. Thus higher valued HQLAs should be held. On the other hand, when banks hold less HQLAs, they should decrease their NCOs that may lead to higher LCRs.

### 5.1.3. Conclusions about numerical results for LCRs

We approximate the Basel III standard, LCR, that is a measure of asset liquidity for global EMERG banking data mentioned earlier. This is a challenging task given the nature of the data available and the ever-changing nature of Basel III liquidity regulation. In the light of the determined results, our analysis gives us a new understanding of the problem of approximating liquidity risk measures. From Table 1, we observe that from Q209 to Q412 there was a steady increase in the LCR. This is probably due to banks holding more liquid assets and restricting cash outflows and risky activities.

In this paragraph, we highlight how our research on approximating Basel III and traditional liquidity risk measures has advanced the knowledge in this field of endeavor. For both Class I and II banks, our research approximates LCRs for a large diversity of banks for an extended

that amounts to determining the value  $J^*$ , given by

$$J^* = \inf_{g \in \mathcal{G}_A} J(g),$$

and the optimal control law  $g^*$ , if it exists,

$$g^* = \arg \min_{g \in \mathcal{G}_A} J(g) \in \mathcal{G}_A.$$

### 3.2. Optimal bank LCRs in the simplified case

In this section, we determine a solution to Problem 3.2 in the case where the term  $[t_0, t_1]$  is fixed. In order to find the optimal control processes, we use the dynamic programming algorithm for stochastic optimization where we consider an appropriate Hamilton-Jacobi-Bellman equation (HJBE). In the sequel, we assume that the optimal control laws exist, with the objective function,  $J$ , given by (16) being continuous twice-differentiable. Then a combination of integral calculus and Itô's formula (see, for instance, [15]) shows that the value function  $v$  satisfies (20) and (21).

Consider the simplified system (13) for the LCR problem with the admissible class of control laws,  $\mathcal{G}_A$ , given by (14) but with  $\mathcal{X} = \mathbb{R}$  (compare with [10]). In this section, we have to solve

$$J^* = \inf_{g \in \mathcal{G}_A} J(g), \quad (17)$$

$$J(g) = \mathbb{E} \left[ \int_{t_0}^{t_1} \exp(-r^f(l - t_0)) [b^2(u_t^2) + b^3(x_t)] dt + \exp(-r^f(t_1 - t_0)) b^1(x(t_1)) \right], \quad (18)$$

where  $b^1 : \mathbb{R} \rightarrow \mathbb{R}^+$ ,  $b^2 : \mathbb{R} \rightarrow \mathbb{R}^+$  and  $b^3 : \mathbb{R}^+ \rightarrow \mathbb{R}^+$  are all Borel measurable functions. For the simplified case, the optimal cost function (17) is determined with the simplified cost function,  $J(g)$ , given by (18). In this case, assumptions have to be made in order to find a solution for the optimal cost function,  $J^*$  (compare with [10]). Next, we state an important result about optimal bank coverage ratios in the simplified case.

#### Theorem 3.3. (Optimal Bank LCRs in the Simplified Case)

Suppose that  $g^{2*}$  and  $g^{3*}$  are the components of the optimal control law,  $g^*$ , that deal with the optimal cash injection rate,  $u^{2*}$ , and optimal HQLA allocation,  $\pi^{k*}$ , respectively. Consider the nonlinear optimal stochastic control problem for the simplified LCR system (13) formulated in Problem 3.2. Suppose that the following assumptions hold.

availability of more suitable data of sufficient granularity as well as improved extrapolation and interpolation techniques.

We have already made several contributions in support of the endeavors outlined in the previous paragraph. For instance, our journal article [12] deals with issues related to liquidity risk and the financial crisis. Also, the role of information asymmetry in a subprime context is related to the main hypothesis of the book [14].

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